

# Semantic information and artificial intelligence

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**Abstract** For a computational system to be intelligent, it should be able to perform, at least, basic deductions. Nonetheless, since deductions are, in some sense, equivalent to tautologies, it seems that they do not provide new information. The present article proposes a measure the degree of semantic informativity of valid deductions in a dynamic setting. Concepts of coherency and relevancy, displayed in terms of insertions and deletions on databases, are used to define semantic informativity. In this way, the article shows that a solution to the problem about the informativity of deductions provides a heuristic principle to improve the deductive power of computational systems.

## 1 Introduction

For Aristotle, “every belief comes either through syllogism or from induction” (Aristotle (1989)). From that, we can infer that every computational system that aspires to exhibit characteristics of intelligence needs to have deductive as well as inductive abilities. With respect to the latter, there are theories that explain why induction is important for artificial intelligence; for instance, Valiant’s probably approximately correct semantics of learning (Valiant (1984, 2008)). Nonetheless, as far as the former is concerned, we have a problem first observed by Hintikka (Hintikka (1973)), which can be stated in the following way:

1. A deduction is valid if, and only if, the conjunction of its premisses, says  $\phi_1, \dots, \phi_n$ , implies its conclusion,  $\psi$ .
2. In this case,  $\phi_1 \wedge \dots \wedge \phi_n \rightarrow \psi$  is a tautology, i.e., valid deductions are equivalent to propositions without information.
3. Therefore, deductions are uninformative.

This was called by Hintikka the *scandal of deduction*. This is a scandal not only because it contradicts Aristotle’s maxim that deductions are important for obtaining beliefs, but, mainly, in virtue of the fact that we actually obtain information via deductions. For this and other reasons, Floridi has proposed a theory of strong seman-

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tic information in which semantic information is true well-formed data (Cf. Floridi (2004)). From this standpoint, Floridi is capable of explaining why some logical formulas are more informative than others. If we want to explain why deductions, not only propositions, are important for knowledge acquisition and intelligent processing, we cannot, however, apply Floridi's theory. The main reason is that it was designed to measure the static semantic information of the data expressed by propositions. In contrast, knowledge acquisition and intelligence are dynamic phenomena, associated in some way to the flow of information.

In the present work, we propose to overcome that limitation by defining a measure of semantic information in Floridi's sense, but in the context of a dynamic perspective about the logical features of databases associated to valid deductions (Section 2). We will restrict ourselves to first-order deductions and will adopt a semantic perspective about them, which means that deductions will be analyzed in terms of structures. There is two reasons for that choice. The first is that the scandal of deduction is usually conceived in terms of structures associated to valid deductions (Cf. Sequoia-Grayson (2008)). In other words, we have a problem with the semantic informativity of deductions. The second reason is technical: databases are finite structures and so there is, in general, no complete deductive first-order logical system for finite structures (Cf. Ebbinghaus and Flum (1999)).

Besides that, we impose to ourselves the methodological constraint that a good approach of semantic informativity should be robust enough to be applicable to real computational systems. More specifically, we will look for a solution that enables to link semantic information and artificial intelligence. Because of that, we propose to measure the degree of semantic informativity of deductions as a dynamic phenomenon, based on certain explicit definitions of insertions and deletions on databases. In that context, the concepts of coherency and relevancy will be explained by the operations of insertion and deletion (Section 3), and so semantic informativity will be conceived in terms of relevancy and coherency (Section 4). This approach leads us to a solution the scandal of deduction (Section 4). Moreover, using straightforward definitions, we shows that our definition of semantic informativity provides an immediate heuristic principle to improve the deductive power of computational systems in semantic terms.

## 2 Databases

We intend to analyze the semantic informativity obtained via deductions. According to Floridi, semantic information is true well-defined data (Floridi (2011)). As far as logic is concerned, we can say that data is in some way expressed by propositions. In general, deductions are compounded of two or more propositions. Thus, we need indeed to consider databases, because they are just organized collections of data (Cf. Kroenke and Auer (2007)). From a logical point of view, the usual notion of database can, in turn, be understood in terms of the mathematical concept of structure.

**Definition 2.1** A database is a pair  $D = (A, T)$  where  $A$  is a finite first-order structure over a signature  $S$  and  $T$  is a correct (all propositions in  $T$  are true in  $A$ ) finite first-order theory about  $A$ .

**Example 2.1** Let  $D_1 = (A_1, T)$  be a database with signature  $S = (\{s, l, a\}, \{C, E, H\})$ , for  $s = \text{``São Paulo''}$ ,  $l = \text{``London''}$ ,  $a = \text{``Avenida Paulista''}$ ,  $C = \text{``City''}$ ,  $E = \text{``Street''}$  and  $H = \text{``To have''}$ , such that  $A_1 = (\{\bar{s}, \bar{l}, \bar{a}\}, \{\bar{s}, \bar{l}, \bar{a}_a, \{\bar{s}, \bar{l}\}_C, \{\bar{a}\}_E, \{(\bar{s}, \bar{a}), (\bar{l}, \bar{a})\}_H\})$  and  $T = \{\forall x(Cx \rightarrow \exists yHxy), \forall x(Cx \vee Ex), \neg El, Cs\}$ .

**Remark 2.1** In the example 2.1 we have used  $\ulcorner \alpha \urcorner = \beta$  to mean that the symbol  $\beta$  is a formal representation of the expression  $\alpha$ . Besides,  $X_\beta$  is the interpretation of  $\beta$  in the structure  $A$  and we use a bar above letters to indicate individuals of the domain of  $A$ .

The fact that theory  $T$  is correct with respect to  $A$  does not exclude, however, the possibility that our database does not correspond to reality. In the example 2.1, it is true in  $A$  that  $Hla \wedge Ea$ , in words, it is true in  $A$  that London has a city called Avenida Paulista, which, until date of the present paper, it is not true. The theory  $T$  represents the fundamental facts of the database that are took as true, that is to say, they are the *beliefs* of the database. It is important to observe that  $T$  may not be complete about  $A$ , it is possible that not all true propositions about  $A$  are in  $T$ ; example 2.1 shows this.

We turn now to the dynamics of changes in databases that will permit us to measure semantic informativity. We propose that these changes in the structure of databases should preserve the truth proposition of their theories through operations that we call *structural operations*. The first structural operation consists in put possibly new objects in the structure of the given database and to interpret a possibly new symbols in terms of these objects.

**Definition 2.2** Let  $D = (A, T)$  be a database over a signature  $S$ . An insertion of the  $n$ -ary symbol  $\sigma \in S'$  in  $D$  is a database  $D' = (A', T)$  where  $A'$  is an structure over  $S' = S \cup \{\sigma\}$  with the following properties:

1.  $A'(\tau) = A(\tau)$  for all  $\tau \neq \sigma$  such that  $\tau \in S$ ;
2. If  $n = 0$ , then  $A' = A \cup \{a\}$  and  $A'(\sigma) = a$ , provided that, for all  $\phi \in T$ ,  $A' \models \phi$ ;
3. If  $n > 0$ , then  $A' = A \cup \{a_1, \dots, a_n\}$  and  $A'(\sigma) = A(\sigma) \cup \{(a_1, \dots, a_n)\}$ , provided that, for all  $\phi \in T$ ,  $A' \models \phi$ .

**Example 2.2** Let  $D_1 = (A, T)$  be the database of the example 2.1. The database  $D_2 = (A_2, T)$  with signature  $S' = S \cup \{b\}$ , where  $b = \text{``Shaftesbury Avenue''}$ , and  $A_2 = (\{\bar{s}, \bar{l}, \bar{a}\}, \{\bar{s}, \bar{l}, \bar{a}_a, \bar{a}_b, \{\bar{s}, \bar{l}\}_C, \{\bar{a}\}_E, \{(\bar{s}, \bar{a}), (\bar{l}, \bar{a})\}_H\})$  is an insertion of  $b$  in  $D$ . On the other hand,  $D_3 = (A_3, T)$  is an insertion of  $E$  in  $D_3$  where  $A_3 = (\{\bar{s}, \bar{l}, \bar{a}, \bar{b}\}, \{\bar{s}, \bar{l}, \bar{a}_a, \bar{a}_b, \{\bar{s}, \bar{l}\}_C, \{\bar{a}, \bar{b}\}_E, \{(\bar{s}, \bar{a}), (\bar{l}, \bar{a})\}_H\})$  is an  $S'$ -structure. Nonetheless, for  $A^* = (\{\bar{s}, \bar{l}, \bar{a}, \bar{b}\}, \{\bar{s}, \bar{l}, \bar{a}_a, \bar{b}_b, \{\bar{s}, \bar{l}\}_C, \{\bar{a}\}_E, \{(\bar{s}, \bar{a}), (\bar{l}, \bar{a})\}_H\})$ , an  $S'$ -structure, we have that  $D^* = (A^*, T)$  is not an insertion of  $b$  in  $D_1$  because in this case  $A^* \not\models \forall x(Cx \vee Ex)$ .

The example 2.2 shows that it is not necessary to introduce a new object in the structure of the database to make an insertion (Cf. database  $D_2$ ), it is sufficient to add a possibly new element in the interpretation of some symbol. On the other hand, it also shows that it is not sufficient to introduce a new object in the structure of the database to make an insertion (Cf. database  $D^*$ ), it is necessary to guarantee that the beliefs of the database are still true in the new structure.

The second structural operation consists in removing possibly old objects in the structure of the database and to interpret a possibly new symbol in terms of the remaining objects in the database.

**Definition 2.3** *Let  $D = (A, T)$  be a database over a signature  $S$ . A deletion of the  $n$ -ary symbol  $\sigma \in S$ ,  $S - \{\sigma\} \subseteq S' \subseteq S$ , from  $D$  is a database  $D' = (A', T)$  where  $A'$  is an structure over  $S'$  with the following properties:*

1.  $A'(\tau) = A(\tau)$  for all  $\tau \neq \sigma$  such that  $\tau \in S$ ;
2. If  $n = 0$ ,  $A - \{A(\sigma)\} \subseteq A' \subseteq A$  and  $A'(\sigma) \in A'$ , provided that, for all  $\phi \in T$ ,  $A' \models \phi$ ;
3. If  $n > 0$ ,  $A - \{a_1, \dots, a_n\} \subseteq A' \subseteq A$  and  $A'(\sigma) = A(\sigma) - \{a_1, \dots, a_n\}$ , provided that, for all  $\phi \in T$ ,  $A' \models \phi$ .

**Example 2.3** *Let  $D_1 = (A, T)$  be the database of the example 2.1. The database  $D'_2 = (A'_2, T)$  with signature  $S$  and  $A'_2 = (\{\bar{s}, \bar{l}, \bar{a}\}, \bar{a}_s, \bar{l}_l, \bar{a}_a, \{\bar{s}, \bar{l}\}_C, \{\bar{a}\}_E, \{(\bar{s}, \bar{a}), (\bar{l}, \bar{a})\}_H)$  is a deletion of  $s$  in  $D$ . On the other hand,  $D'_3 = (A'_3, T)$  is a deletion of  $H$  in  $D'_2$  where  $A'_3 = (\{\bar{s}, \bar{l}, \bar{a}\}, \bar{a}_s, \bar{l}_l, \bar{a}_a, \{\bar{s}, \bar{l}\}_C, \{\bar{a}\}_E, \{(\bar{l}, \bar{a})\}_H)$  is a  $S$ -structure. Nonetheless, for  $A'_4 = (\{\bar{l}, \bar{a}\}, \bar{a}_s, \bar{l}_l, \bar{a}_a, \{\bar{l}\}_C, \{\bar{a}\}_E, \{(\bar{l}, \bar{a})\}_H)$ , an structure over the signature  $S$ , we have that  $D'_4 = (A'_4, T)$  is not a deletion of  $C$  in  $D'_2$  because in this case, in despite of  $A'_4 \models \phi$  for  $\phi \in T$ , we have that  $D'_4(H) \neq D'_2(H)$ . Note, however, that  $D'_4$  is a deletion of  $C$  in  $D'_3$ .*

Example 2.3 illustrates that the restriction  $A - \{a_1, \dots, a_n\} \subseteq A' \subseteq A$  means that we can delete at most just the elements of the domain that we remove in some way from the interpretation of the symbol that we are considering in the deletion.

Insertions and deletions on databases are well known primitive operations (Cf. Kroenke and Auer (2007)). Nevertheless, to the best of our knowledge, they have being conceived as undefined notion. Here we have proposed, however, a logical conception about databases and we have defined explicitly the operations of insertion and deletion sufficient in order to analyze the importance of semantic information to artificial intelligence. In Araújo (2014), a more strict notion of structural operation is given.

### 3 Coherency and relevancy

In this section, we propose a dynamic perspective about coherency and relevancy. This approach will permit us to evaluate how many structural operations a propo-

sition requires to become true. We will use these concepts to define the semantic informativity in the next section.

**Definition 3.1** An update  $\bar{D}$  of an  $S$ -database  $D$  is a finite or infinite sequence  $\bar{D} = (D_i : 0 < i \leq \omega)$  where  $D_1 = D$  and each  $D_{i+1}$  is a insertion or deletion in  $D_i$ . An update  $\bar{D}$  of  $D$  is coherent with a proposition  $\phi$  if  $\bar{D} = (D_1, D_2, \dots, D_n)$  and  $A_n \models \phi$ ; otherwise,  $\bar{D}$  is said to be incoherent with  $\phi$ .

**Example 3.1** Let  $D_1$  be the database of the example 2.1 and  $D_2$  be the databases of the example 2.2. The sequence  $\bar{D} = (D_1, D_2)$  is an update of  $D$  coherent with  $Eb$  and  $Hlb$ . Let  $D_1$  be the database of the example 2.1 and  $D'_2, D'_3$  and  $D'_4$  be the databases of the example 2.3. The sequence  $\bar{D}' = (D_1, D'_2, D'_3, D'_4)$  is an update of  $D$  coherent with  $Es \wedge \neg Hsa$  but not with  $s = a$  because the last proposition is false in  $A'_4 = (\{\bar{l}, \bar{a}\}, \bar{a}_s, \bar{l}_l, \bar{a}_a, \{\bar{l}\}_C, \{\bar{a}\}_E, \{(\bar{l}, \bar{a})\}_H)$ .

In other words, an update for a proposition  $\phi$  is a sequence of changes in a given database that produces a structure in which  $\phi$  is true. In this way, we can measure the amount of coherency of propositions.

**Definition 3.2** Let  $\bar{D} = (D_1, D_2, \dots, D_n)$  be an update of the database  $D$ . If  $\bar{D}$  is coherent with  $\phi$ , we define the coherency of  $\phi$  with  $\bar{D}$  by

$$H_{\bar{D}}(\phi) = \frac{\min\{m \leq n : A_m \models \phi\}}{\sum_{i=1}^m i}$$

but if  $\bar{D}$  is incoherent with  $\phi$ , then

$$H_{\bar{D}}(\phi) = 0.$$

A proposition  $\phi$  is said to be coherent with the database  $D$  if  $H_{\bar{D}}(\phi) > 0$  for some update  $\bar{D}$ , otherwise,  $\phi$  is incoherent with  $D$ .

**Remark 3.1** In the definition of coherency the denominator  $\sum_{i=1}^m i$  is used in to order to normalize the definition (the coherency is a non-negative real number smaller than or equal to 1).

**Example 3.2** The coherence of  $Eb$  and  $Hlb$  with the update  $\bar{D}$  of the example 3.1 is the same  $2/3$ , i.e.,  $H_{\bar{D}}(Eb) = H_{\bar{D}}(Hlb) \approx 0.66$  and so  $H_{\bar{D}}(Eb \wedge Hlb) = H_{\bar{D}}(Eb \vee Hlb) \approx 0.66$ . On the other hand, with respect to the coherence of  $Es$ ,  $\neg Hsa$  and  $\neg s = a$  and with the update  $\bar{D}'$  of the example 3.1, we have  $H_{\bar{D}'}(Es) \approx 0.66$ ,  $H_{\bar{D}'}(\neg Hsa) = 0.4$ ,  $H_{\bar{D}'}(s = a) = 0$  and so  $H_{\bar{D}'}(Es \wedge \neg Hsa) = 0.4$  but  $H_{\bar{D}'}(Es \wedge s = a) = 0$ .

The example 3.2 exhibits that, given an update, we can have different propositions with different coherency, but we can have different propositions with the same coherency as well. The fact that so  $H_{\bar{D}}(Eb) = H_{\bar{D}}(Hlb) = H_{\bar{D}}(Eb \wedge Hlb) \approx 0.66$  shows that concept of coherency is *not* a measure of the complexity of propositions. It seems natural to think that  $Eb \wedge Hlb$  is in some sense more complex than  $Eb$  and  $Hlb$ . Here we do not have this phenomena. Moreover, the fact that

$H_{\bar{D}}(Eb \wedge Hlb) = H_{\bar{D}}(Eb \vee Hlb) \approx 0.66$  makes clear that, since some propositions have a given coherency, many others will have the same coherency. Another interesting point is that  $H_{\bar{D}'}(Es) > H_{\bar{D}'}(\neg Hsa)$  but  $H_{\bar{D}'}(\neg Hsa) = H_{\bar{D}}(Eb \wedge Hlb) \approx 0.33$ . This reflects the fact that updates are sequences. First, we had made  $Es$  coherent with  $\bar{D}'$ , and later  $\neg Hsa$  we made coherent with  $\bar{D}$ . When  $\neg Hsa$  is coherent with  $\bar{D}'$  there is nothing more to be done, as far as the conjunction  $Es \wedge \neg Hsa$  is concerned.

These remarks show that our approach is very different from the one given in (Cf. D'Agostino and Floridi (2009)). It is not an analysis of some concept of complexity associated to semantic information. In Araújo (2014), we do an analysis of informational complexity similar to one presented here about coherency, but this two concepts are different. In further works, we will examine the relation between them. For now, we are interested in artificial intelligence. With respect to that, we can obtain an important result in the direction of a solution to the scandal of deduction.

**Proposition 3.1** *For every database  $D = (A, T)$  and update  $\bar{D}$  coherent with  $\phi$ ,  $H_{\bar{D}}(\phi) = 1$  for every  $\phi$  such that  $A \models \phi$ . In particular, for  $\phi$  a tautology in the language of  $D$ ,  $H_{\bar{D}}(\phi) = 1$ , but if  $\phi$  is not in the language of  $D$ ,  $0 < H_{\bar{D}}(\phi) < 1$ . In contrast, for every contradiction  $\psi$  in any language,  $H_{\bar{D}}(\psi) = 0$  for every update  $\bar{D}$  of  $D$ .*

In virtue of our focus in this paper is conceptual, we will not provide proofs here (Cf. Araújo (2014)). By now, we only observe that if a tautology has symbols different from the ones in the language of the database, it will be necessary to make some changes in order to make that tautology become true. In contrast, a proposition is incoherent with a database when there is no way to change it in order to become the proposition true and, for this reason, contradictions are never coherent.

We turn now to the concept of relevancy. For that, let us introduce a notation. Consider  $(\phi_1, \phi_2, \dots, \phi_n)$  a valid deduction of formulas over the signature  $S$  whose premisses are the set  $\Gamma = \{\phi_1, \phi_2, \dots, \phi_m\}$  and its conclusion is  $\phi = \phi_n$ . We represent this deduction by  $\Gamma\{\phi\}$ .

**Definition 3.3** *Let  $\bar{D} = (D_1, \dots, D_n)$  be an update of the  $S$ -database  $D = (A, T)$  coherent with  $\phi$ . The relevant premisses of the deduction  $\Gamma\{\phi\}$  with respect to  $\bar{D}$  are the premisses that are true in  $D_n$  but are not logical consequences of  $T$ , i.e., the propositions in the set  $\bar{D}(\Gamma)$  of all  $\psi \in \Gamma$  for which  $D_n \models \psi$  but  $T \not\models \psi$ .*

**Example 3.3** *Let  $\bar{D}'' = (D_1)$ . Then,  $\bar{D}''(\{Ea\}\{\exists xEx\}) = \{Ea\}$ . Now let us consider a more complex example. Let  $\bar{D} = (D_1, D_2)$  be the update of 3.1. In this case,  $\bar{D}(\{\forall x(Cx \rightarrow \neg Ex), Cb\}\{\neg Eb\})$  is not defined because  $\neg Eb$  is false in  $A_2 = (\{\bar{s}, \bar{l}, \bar{a}\}, \bar{s}_s, \bar{l}_l, \bar{a}_a, \bar{a}_b, \{\bar{s}, \bar{l}\}_C, \{\bar{a}\}_E, \{(\bar{s}, \bar{a}), (\bar{l}, \bar{a})\}_H)$ . Nonetheless, consider the new update  $\bar{D}''' = (D_1, D_2, D_3, D_4, D_5)$  such that  $D_3$  is the insertion in example 2.2,  $A_4 = (\{\bar{s}, \bar{l}, \bar{a}, \bar{b}\}, \bar{s}_s, \bar{l}_l, \bar{a}_a, \bar{b}_b, \{\bar{s}, \bar{l}\}_C, \{\bar{a}, \bar{b}\}_E, \{(\bar{s}, \bar{a}), (\bar{l}, \bar{a})\}_H)$  and  $A_5 = (\{\bar{s}, \bar{l}, \bar{a}, \bar{b}\}, \bar{s}_s, \bar{l}_l, \bar{a}_a, \bar{b}_b, \{\bar{s}, \bar{l}\}_C, \{\bar{a}\}_E, \{(\bar{s}, \bar{a}), (\bar{l}, \bar{a})\}_H)$ . Then,  $\bar{D}'''(\{\forall x(Cx \rightarrow \neg Ex), Cb\}\{\neg Eb\}) = \{\forall x(Cx \rightarrow \neg Ex)\}$ .*

In our definition of relevant premisses, we have adopted a semantic perspective oriented to conclusion of deductions: the relevancy of the premisses of a deductions

are determined according to an update in which its conclusion is true. Example 3.3 illustrates that point, because it is only possible to evaluate the relevancy of  $\{\forall x(Cx \rightarrow \neg Ex), Cb\}\{\neg Eb\}$  in an update like  $\bar{D}'$  in which the conclusion  $\neg Eb$  is true. Another point to be noted is that we have chosen a strong requirement about what kind of premises could be relevant: the relevant premises are just the non-logical consequences of our believes.

**Definition 3.4** *Let  $D$  be an  $S$ -database. If  $\bar{D}$  is an update of  $D$  coherent with  $\phi$ , the relevancy  $R_{\bar{D}}(\Gamma)$  of the deduction  $\Gamma\{\phi\}$  in  $\bar{D}$  is the cardinality of  $\bar{D}(\Gamma)$  divided by the cardinality of  $\Gamma$ , i.e.,*

$$R_{\bar{D}}(\Gamma) = \frac{|\bar{D}(\Gamma)|}{|\Gamma|},$$

*but, if  $\bar{D}$  is incoherent with  $\phi$ , then  $R_{\bar{D}}(\Gamma) = 0$ .*

**Example 3.4** *We have showed in example 3.3 that  $R_{\bar{D}''}(\{Ea\}\{\exists xEx\}) = 1$  and  $R_{\bar{D}'''}(\{\forall x(Cx \rightarrow \neg Ex), Cb\}\{\neg Eb\}) = 0.5$ .*

In the example above,  $R_{\bar{D}'''}(\{\forall x(Cx \rightarrow \neg Ex), Cb\}\{\neg Eb\}) = 0.5$  show us that we can have valid deductions with non-null relevancy in extended languages. Nonetheless, the fact  $R_{\bar{D}''}(\{Ea\}\{\exists xEx\}) = 1$  shows that is not necessary to consider extended languages to find deductions with non-null relevancy. Thus, we have a result that will be central to our solution of the scandal of deduction.

**Proposition 3.2** *For every database  $D = (A, T)$ , update  $\bar{D}$  of  $D$  and deduction  $\Gamma\{\phi\}$ , if  $T$  is a complete theory of  $A$  or  $\Gamma = \emptyset$ , then  $R_{\bar{D}}(\Gamma) = 0$ . In particular, tautologies and contradictions have null relevancy.*

Therefore, deductions can be relevant only when we do not have a complete theory of the structure of the database. Moreover, as deductions, isolated logical facts (tautologies and contradictions) have no relevance. This means that we have at hand a deductive notion of relevancy.

## 4 Semantic informativity and artificial intelligence

Having at hand the dynamic concepts of coherence and relevance, now it seems reasonable to say that the more coherent the conclusion of a valid deduction is the more informative it is, but the more relevant its premises are the more information they provide. We use this intuition to define the semantic informativity of valid deductions.

**Definition 4.1** *The semantic informativity  $I_{\bar{D}}(\Gamma\{\phi\})$  of a valid deduction  $\Gamma\{\phi\}$  in the update  $\bar{D}$  of the database  $D$  is defined by*

$$I_{\bar{D}}(\Gamma\{\phi\}) = R_{\bar{D}}(\Gamma)H_{\bar{D}}(\phi).$$

The idea behind the definition of semantic informativity of a valid deduction  $\Gamma\{\phi\}$  is that  $I_{\bar{D}}(\Gamma\{\phi\})$  is directly proportional to the relevance of its premises  $\Gamma$  and to the coherency of its conclusion  $\phi$ . Given  $\Gamma\{\phi\}$  and an update  $\bar{D}$  of  $D$ , if we have  $R_{\bar{D}}(\Gamma) = 0$  or  $H_{\bar{D}}(\phi) = 0$ , then the semantic informativity of  $\Gamma\{\phi\}$  is zero, it does not matter how  $\Gamma\{\phi\}$  is. Now, if  $H_{\bar{D}}(\phi) = 0$ , then, by definition,  $R_{\bar{D}}(\Gamma) = 0$ . Thus, if the computational system, whose database is  $D$ , intends to evaluate  $I_{\bar{D}}(\Gamma\{\phi\})$  for some update  $\bar{D}$ , it should look for a  $\bar{D}$  coherent with  $\phi$ , i.e., a  $\bar{D}$  for which  $H_{\bar{D}}(\phi) > 0$ . In other words, our analysis of the semantic informativity is oriented to the conclusion of valid deductions - as we did with respect to relevancy.

**Example 4.1** *Given the updates  $\bar{D}''$  and  $\bar{D}'''$  of the example 3.3. Then,  $I_{\bar{D}''}(\{Ea\}\{\exists xEx\}) = 1 \cdot 1 = 1$  and  $I_{\bar{D}'''}(\{\forall x(Cx \rightarrow \neg Ex), Cb\}\{\neg Eb\}) = 0.5 \cdot 5/15 \approx 0.17$ .*

In the definition of  $I_{\bar{D}}(\Gamma\{\phi\})$  the relevancy of the premisses,  $R_{\bar{D}}(\Gamma)$ , is a factor of the coherency of the conclusion,  $H_{\bar{D}}(\phi)$ . For that reason, if a computational systems intends to evaluate the semantic informativity of a proposition  $\phi$ , it should measure  $H_{\bar{D}}(\phi)$  and, then, multiply it by its relevancy  $R_{\bar{D}}(\{\phi\})$ . Hence, the semantic informativity of a proposition  $\phi$  can be conceived as a special case of the informativity of the valid deduction  $\{\phi\}\{\phi\}$ .

**Definition 4.2** *The semantic informativity  $I_{\bar{D}}(\phi)$  of a proposition  $\phi$  in the update  $\bar{D}$  of the database  $D$  is defined by*

$$I_{\bar{D}}(\phi) = I_{\bar{D}}(\{\phi\}\{\phi\}).$$

**Example 4.2** *Considering the update  $\bar{D}''$  of example 3.3, we have that  $I_{\bar{D}''}(Ea) = I_{\bar{D}''}(\exists xEx) = 1$  but  $I_{\bar{D}''}(Ea \rightarrow \exists xEx) = 0$ . If we consider the update  $\bar{D}'''$  of example 3.3, we have that also have that  $I_{\bar{D}'''}(\forall x(Cx \rightarrow \neg Ex) \wedge Cb) \rightarrow \neg Eb = 0$ , but  $I_{\bar{D}'''}(\forall x(Cx \rightarrow \neg Ex)) = 1$ ,  $I_{\bar{D}'''}(Cb) = 0$  and  $I_{\bar{D}'''}(\neg Eb) = 0.4$ .*

Example 4.2 shows that semantic informativity measures how many structural operations we do in order to obtain the semantic information of a proposition. It is for that reason that  $I_{\bar{D}'''}(Cb) = 0$ , false well-defined data is not semantically informative; it should be true. In other words, it is a measure of semantic information in Floridi's sense (Cf. Floridi (2011)). From this, we can solve Hintikka's scandal of deduction.

**Proposition 4.1** *For every valid deduction  $\psi_1, \dots, \psi_n \vdash \phi$  in the language of  $D$ ,  $I_{\bar{D}}((\psi_1 \wedge \dots \wedge \psi_n) \rightarrow \phi) = 0$  for every update  $\bar{D}$ . Nonetheless, if  $\psi_1, \dots, \psi_n \vdash \phi$  is not in the language of  $D$ ,  $I_{\bar{D}}((\psi_1 \wedge \dots \wedge \psi_n) \rightarrow \phi) > 0$  for  $\bar{D}$  coherent with  $(\psi_1 \wedge \dots \wedge \psi_n) \rightarrow \phi$ .*

This proposition is a solution to the scandal of deduction in two different senses. First, it shows that we can have an informative valid deduction  $\{\psi_1, \dots, \psi_n\}\{\phi\}$  whose associated conditional  $\psi_1, \dots, \psi_n \rightarrow \phi$  is uninformative, for example, the one given in example 4.1. Second, it shows that it is not completely true that tautologies are always uninformative. When we need to interpret new symbols, we have some



semantic information, notably, the one sufficient to perceive that we have a true proposition - this is a natural consequence of our approach.

From this standpoint, we are going to make a simple, but important, remark to establish a relationship between semantic informativity and artificial intelligence.

Given an update  $\bar{D} = (D_1, \dots, D_n)$  of  $D = (A, T)$  and a deduction  $\{\phi\}\{\phi\}$ , either  $R_{\bar{D}}(\{\phi\}) = 0$  or  $R_{\bar{D}}(\{\phi\}) = 1$ . If  $R_{\bar{D}}(\{\phi\}) = 0$ , then either  $T \models \phi$  or  $D_n \not\models \phi$ . If  $T \models \phi$ , then there is an update  $\bar{D}'$  of  $D$  such that  $H_{\bar{D}'}(\phi) = 1$ , notably,  $\bar{D}' = (D)$ . If  $D_n \not\models \phi$ , then  $H_{\bar{D}'}(\phi) = 0$ . Finally, if  $R_{\bar{D}}(\{\phi\}) = 1$ , then  $T \not\models \phi$  as well as  $D_n \models \phi$  and so  $I_{\bar{D}}(\phi) = H_{\bar{D}}(\phi) > 0$ . Therefore, we conclude that the relevancy of a proposition does not determine its coherency. On the other hand, if  $H_{\bar{D}}(\phi) = 0$ , then  $R_{\bar{D}}(\{\phi\}) = 0$ , but if  $H_{\bar{D}}(\phi) > 0$ , this neither necessarily imply that either  $R_{\bar{D}}(\{\phi\}) = 1$  nor  $R_{\bar{D}}(\{\phi\}) = 0$ , because this depends whether  $T \models \phi$ . Hence, we also conclude that the coherency of a proposition does not determine its relevancy too. Combining this two conclusions we obtain a general conclusion: the semantic informativity of propositions cannot be determined by its coherency or relevancy alone. This reinforces our definition of semantic informativity. It is not an arbitrary definition, in fact semantic informativity is a relationship between both, coherence and relevancy. What is the moral for artificial intelligence?

In the studies of pragmatics (a linguistics' area of research), Wilson and Sperber formulated two principles about relevant information in human linguistic practice:

"Relevance may be assessed in terms of cognitive effects and processing effort: (a) other things being equal, the greater the positive cognitive effects achieved by processing an input, the greater the relevance of the input to the individual at that time; (b) other things being equal, the greater the processing effort expended, the lower the relevance of the input to the individual at that time." Wilson and Sperber (2004)[p.608]

Our general conclusion that semantic informativity of propositions cannot be determined by its coherency or relevancy alone shows that the two Wilson and Sperber's principles (a) and (b) are in fact parts of one general principle associated to semantic information. Let us put that in precise terms.

**Definition 4.3** *The changes that a proposition  $\phi$  requires are the structural operations, insertions and deletions, that generate an update  $\bar{D}$  of a given database  $D = (A, T)$  coherent with  $\phi$ . A proposition  $\phi$  is new if  $\phi$  is not false in  $A$  and is not a consequence of the theory  $T$  of the database  $D = (A, T)$ .*

**Proposition 4.2** *The less changes a new proposition requires, the more informative it is.*

Proposition 4.2 is a direct consequence of definition 4.2. As we have showed that our definition 4.2 is not, in turn, arbitrary, this means that 4.2 is not arbitrary. 4.2 is a reformulation of the Wilson and Sperber's principle (b) above, but it is important to note the differences between them. Wilson and Sperber's principle (b) is an empirical matter under discussion among linguistics (Cf. Wilson and Sperber (2004)). The proposition 4.2 is a reformulation of our definition of semantic informativity. Using the same strategy, we can also obtain a formal version of Wilson and Sperber's principle (a).

**Definition 4.4** *If a valid deduction  $\Gamma\{\phi\}$  has non-null relevancy in a given update  $\bar{D}$  and its conclusion  $\phi$  is new, then the results that it produces are its relevant premisses and its conclusion, i.e.,  $\bar{D}(\Gamma) \cup \{\phi\}$ , but if  $\phi$  is not new, then the results that it produces are just its relevant premisses  $\bar{D}(\Gamma)$ .*

**Proposition 4.3** *The more results a valid deduction produces, the more informative it is.*

We can, then, combine these two propositions in an schematic one.

**Proposition 4.4 (Principle of semantic informativity)** *To increase semantic informativity, an intelligent agent, with respect to its database, should perform little changes and produce big results.*

In recent works (Cf. Valiant (2008)), Valiant have argued that one of the most important challenges in artificial intelligence is that of understanding how computational systems that acquire and manipulate commonsense knowledge can be created. With respect to that point, he explains that some of the lessons from his PAC theory is this:

“We note that an actual system will attempt to learn many concepts simultaneously. It will succeed for those for which it has enough data, and that are simple enough when expressed in terms of the previously reliably learned concepts that they lie in the learnable class.”  
Valiant (2008)[p.6]

We can read Valiant’s perspective in terms of the principle of semantic informativity. The simple propositions are the more coherent propositions, the ones that requires little changes in the database. To have enough data is to have propositions sufficient to deduce another propositions and this means that deductions with more results are preferable. Of course, Valiant’s remark relies on PAC, a theory about learnability, not on deductivity. It is necessary to develop further works to make clear the relationship between this two concepts. Since we have designed a concept of semantic informativity implementable in real systems, it seems, however, that the possibility of realizing that is open.

## 5 Conclusion

We have proposed to measure the degree of semantic informativity of deductions by means of dynamic concepts of relevancy and coherency. In an schematic form, we can express our approach in the following way:

$$\text{Semantic informativity} = \text{Relevancy} \times \text{Coherency}.$$

In accordance with this conception, we showed how the scandal of deduction can be solved. Our solution is that valid deductions are not always equivalent to propositions without information. It is important to note, however, that this problem is

not solve in its totality, because here we have analyzed semantic information only from the point of view of relevancy and coherency. Another crucial concept associated to semantic information is the notion of complexity. In Araújo (2014), we treat this subject, but a complete analysis of the relation among semantic information, relevance, coherency and complexity is necessary.

In this respect, we have derived a principle of semantic informativity that, when applied to computational intelligent systems, means that an intelligent agent should make few changes in its database and obtain big results. This seems an obvious observation, but it is not. The expressions “few changes” and “big results” here have a technical sense which opens the possibility of relating semantic information and artificial intelligence in a precise way. Indeed, there is a lot of possible developments to be explored, we would like to indicate three.

The first one is to investigate the connections between semantic informativity and machine learning, specially, with respect to Valiant’s semantic theory of learning (PAC). As in Valiant’s PAC, we could establish probability distributions on the possible updates and delineate goals for them - the principle of semantic informativity could play an important role in this point. Moreover, we can also introduce computational complexity constraints to agent semantic informativity. Thus, it will be possible to analyze how many efficient updates (time and space requirements bounded by a function of the proposition size) are necessary for a given proposition to be coherent with the database. In this way, we will be able, for example, to compare the learnability of the concepts which occur in propositions, in the Valiant’s sense (Cf. Valiant (1984)), with respect to their semantic information.

The second possible line of research is to develop a complete dynamic theory of the semantic informativity by incorporating belief revision in the line of AGM theory (Cf. Alchourrón et al (1985)). In the present paper, the beliefs of the database have been maintained fixed, but a more realistic approach should incorporate revision of beliefs. For example, if we consider distributed systems, the agents probably will have some different beliefs. In this case, it will be necessary to analyze the changes of semantic information, conflicting data and so on.

The last point to be explored, but no less important, is to analyze the relationship between our dynamic perspective about semantic information and other static approaches, mainly, with respect to Floridi’s theory of strong semantic information (Floridi (2004)). It is important to observe that we have proposed a kind of hegelian conception about semantic information, according to which semantic informativity is analyzed in semantic terms, whereas, for example, Floridi’s conception is kantian, in the sense that it analyzes the relationship between propositions and the world in order to understand the transcendental conditions of semantic information (Cf. Floridi (2011)). Our approach seems to be a hegelian turn in the philosophy of information similar to what Brandom did with respect to the philosophy of language (Cf. Brandom (1989)).

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